Strategic Bidding and Spite in Combinatorial Clock Auctions

A Theoretical and Empirical Analysis

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Contents

1 Introduction

This paper investigates why bidders exhibit unexpected bidding behaviors in the Combinatorial Clock Auction (CCA), with a particular focus on demand expansion as prices rise. It proposes that bidders' spitefulness serves as a compelling explanation for such behaviors. Additionally, the paper introduces a novel method to evaluate the existence of spiteful bidding and measure the degree of spitefulness. By combining theoretical modeling with an extended GARP-based identification framework, this study provides a more robust toolset for understanding and detecting spiteful bidding behaviors, offering fresh insights into the strategic dynamics of spectrum auctions.

Auctions have long been a key mechanism for spectrum allocation. In 1994, the FCC introduced auctions as a solution for spectrum allocation, marking a significant step in translating multi-object auction theories into practical application. Over the next two decades, the primary format for spectrum auctions was the Simultaneous Multi-Round Ascending auction (SMRA), a straightforward adaptation of the English auction to a multi-item setting. While the SMRA meets many objectives of multi-item auctions, it has several weaknesses that undermine price discovery and allocation efficiency, including limitations in package discovery and exposure risk [Ausubel et al., 2014]. To address these issues, the combinatorial clock auction (CCA) was proposed as an alternative auction model [Ausubel et al., 2006]. Combining a multi-item clock auction with a sealed-bid Vickrey-Clarke-Groves (VCG) auction, the CCA was initially celebrated for its equilibrium properties, which theoretically incentivize bidders to reveal their true valuations.

Despite these theoretical advantages, practical implementations of the CCA have revealed unexpected bidder behaviors. Ideally, during the clock phase, a bidder's demand should decrease as the clock price increases. However, deviations from this expected behavior—such as demand expansion in the early rounds or unexpected reductions in demand—have been observed. This raises important questions about the robustness of the optimal strategy under the CCA framework. While early research assumed "truth-telling" strategies as optimal in private value settings, more recent studies have challenged this assumption, highlighting the complexities introduced by strategic considerations [Gretschko et al., 2016].

In this paper, the focus is placed on demand expansion, where bidders' demand increases as prices rise. Although the incentives behind this behavior can be attributed to the winner's curse [Thaler, 1988] and the desire to win [Malhotra, 2010], spitefulness offers a more compelling explanation. First, while the spectrum market is a technical market associated with some uncertain valuations, companies usually evaluate the market value in advance based on the covered population and the bandwidth of the spectrum, both of which can be determined beforehand. Second, bidders are not natural persons, and the behavior of companies tends to align more closely with the rationality hypothesis. Therefore, the winner's curse and the desire to win may not be appropriate explanations for this behavior. On the other hand, spiteful bidding behavior—where bidders not only aim to maximize their own payoffs but also seek to increase the costs incurred by their rivals—often emerges in long-term competitive environments, where players strategically aim to weaken their rivals' future capabilities.

This paper contributes to the substantial body of literature on spiteful bidding behaviors in auctions. Levin and Skrzypacz developed a linear-decreasing demand model for the CCA [Levin and Skrzypacz, 2016], later extended by Maarten Janssen to analyze assignment phase behaviors and lexicographic preferences [Janssen and Kasberger, 2019, Janssen and Karamychev, 2016]. Furthermore, theoretical frameworks for spiteful bidding in other auction formats have been explored extensively, including asymmetric spiteful bidder models [Sharma and Sandholm, 2010] and two-dimensional utility models [Brandt et al., 2007]. In the experimental field, Nishimura et al. proposed experimental verifications in second-price sealed-bid and English auctions [Nishimura et al., 2011].

The remainder of this paper is structured as follows: Section 2 provides the background for the research question, including an overview of the CCA mechanism and the telecommunications market structure that incentivizes spiteful bidding. Section 3 constructs a formal model of the auction and bidder utility functions, illustrating how the CCA creates strategic opportunities for spiteful behaviors. Section 4 outlines the identification approach, connecting theoretical predictions with field evidence. Section 5 presents empirical evidence from FCC Auction 107, demonstrating the prevalence of spiteful bidding. Finally, Section 6 concludes the paper, summarizing the findings and highlighting the contributions and implications of this research.

2 Background

2.1 Mechanisms of Combinatorial Clock Auctions (CCA)

The normal form of the combinatorial clock auction (CCA) consists of two main phases—the clock phase and the assignment phase—which are interconnected through activity rules.

The clock phase begins with bidders being assigned eligibility points, determined by financial qualifications (exogenous factors), which enable them to bid at the starting price. In each clock round, bidders specify the quantities they are willing to accept at the current price, thereby expressing their demand. As the clock rounds progress, prices increase based on either a fixed increment or an allowed inter-clock price adjustment [Ausubel et al., 2006]. Eligibility points typically decrease monotonically across rounds, governed by a series of activity rules. In each round, the auctioneer compares the aggregate demand for each category with the available supply, and the clock phase continues as long as any aggregate demand exceeds the supply. The clock phase concludes when no category has excess demand, ideally achieving a "market-clearing" state. In the final round of the clock phase, the auction determines the final prices and allocations, which are linked to the assignment phase through activity rules.

After the clock phase ends, the CCA transitions to the assignment phase, which operates as a single-round sealed VCG auction. In this phase, each bidder submits sealed bids for the bundle they won during the clock phase or for other bundles they are interested in. The assignment phase enforces limitations based on activity rules, typically including the "final-cap" rule, which requires bidders to bid amounts higher than the prices from the final clock round. Once all bids are submitted, the auction solves the winner determination problem by maximizing valuations. However, the winners' payments are calculated differently, adhering to the VCG rule. Specifically, the payment for each winner is based on the second-best allocation, aiming to maximize overall surplus.

The activity mechanism in the CCA aims to prevent strategic bidding behaviors, such as sniping, which have been observed in previous spectrum auctions. The simplest form of activity rules involves monotonic eligibility points assigned to each bidder [Ausubel and Cramton, 2012]. At the start of the clock phase, each bidder is allocated a certain number of eligibility points. Each item is associated with a certain bidding point, which is usually pre-calculated by an evaluation algorithm based on population coverage and future business valuation. Bidders place bids on items using their allocated eligibility points. A bidder cannot bid on a bundle with an aggregate bidding point exceeding the eligibility points they possess. At the end of each round, the eligibility points for each bidder in the next round are calculated based on the aggregate eligibility points they have used in the current round, usually multiplied by a percentage lower than 100% (to ensure monotonicity). To remain active, bidders must meet a minimum activity threshold. However, point-based activity rules are too weak to eliminate strategic bidding behaviors such as "parking" during the clock phase. Parking is a strategic behavior where a bidder places their points on items they do not genuinely desire in the early phases of the auction, aiming to conserve points for sniping later. To address this, Ausubel [Ausubel et al., 2006] proposed an enhancement by linking activity rules to revealed preference. Ausubel and Baranov [Ausubel and Baranov, 2014] further proved that using GARP-based activity rules always permits truthful bidding during the two phases of the CCA.

2.2 Market Dynamics and Strategic Motivations in Spectrum Auctions

The telecommunications industry is a typical oligopoly market, dominated by a small number of competing firms. The concentration of market spectrum allocated through auctions can be measured by the Herfindahl-Hirschman Index (HHI), defined as HHI $= \sum_{i=1}^{N} s_i^2$, where s_i represents the market share of firm i.An example is provided by analyzing the allocation results from FCC Auction 107. Since the bidding points of each license are pre-calculated based on potential market value, the aggregated bidding points of licenses allocated among the winners can be used to estimate the HHI. The resulting HHI was approximately 4092, indicating a highly monopolistic market. In such markets, competitors engage in long-term strategies rather than focusing solely on individual auctions. First, firms have incentives to acquire more valuable bundles than their rivals, securing larger and more valuable market shares for the future. Second, they aim to increase their rivals' costs, potentially constraining their financial budgets, limiting their future development, or restricting their participation in subsequent auctions. Consequently, the behavior observed in spectrum auctions is often more complex than the idealized "truth-bidding" strategy [Gretschko et al., 2016].

3 Theoretical Framework

3.1 Auction Design and Assumptions

Assume there are two bidders, each seeking a share x_i where $x_i \in [0,1]$ and let the bidding function be denoted by β_i. Although the spectrum auction involves both private and common values, it is reasonable to assume that private values dominate in this context. This can be explained by the fact that each bidder has their own business strategy for utilizing the spectrum, which directly impacts the profitability of the license. Consequently, all bidders base their bids on their own valuations while simultaneously adjusting their strategic bidding behavior in response to their rivals' signals.

As described above, the CCA is a hybrid auction that begins with a clock phase, followed by an assignment phase. Although in practice the clock phase typically begins with a reserve price, this reserve price is set low enough to encourage all bidders to participate. To simplify the analysis, it is assumed that the clock phase begins with an initial price of $p_0 = 0$. In each round t, the clock price increases by an increment according to the auction rules. A fixed increment Δp does not affect the analysis to be conducted but provides a clearer research assumption. As the clock phase progresses, the clock price increases and the bidders' demand weakly decreases. The clock phase ends at price $\tilde{p}$ when the aggregate demand is less than or equal to the supply, such that $\sum_i^2 x_i(\tilde{p}) \leq 1$.

The assignment phase begins with the final clock price $\tilde{p}$ and uses VCG auction rules to determine the allocations and payments. The auctioneer chooses the allocation $\mathbf{x} = (x_1, x_2)$ that maximizes $\arg\max_{\mathbf{x}} \sum_i^2 \beta_i(x_i)$, subject to $\sum_i^2 x_i \leq 1$ and $x_i \geq 0$. Bidder i receives a share x_i when placing a bid of $\beta_i(x_i)$. According to the VCG rules, it is also necessary to calculate the optimal allocation profile without bidder i, which solves the maximization $\arg\max_{x_{-i}} \beta_{-i}(x_{-i})$. The optimal hypothetical share of the rival is denoted as $\hat{x}_{-i}$ with the corresponding bid $\beta_{-i}(\hat{x}_{-i})$. The final payment for bidder i is the opportunity cost incurred by the rivals in the absence of bidder i, calculated as $\beta_i(x_i) - (\sum_i^2 \beta_i(x_i) - \beta_{-i}(\hat{x}_{-i}))$. Let $\hat{x}_{-i}$ be denoted as y. The payment for bidder i is given by $\max_y \beta_{-i}(y) - \beta_{-i}(x_{-i})$. In this context, the alternative optimal allocation is $x_{-i} = 1$, allowing the objective function to be simplified to $\max \beta_{-i}(1) - \beta_{-i}(1 - x_i)$.

The CCA uses activity rules to link the clock phase with the assignment phase. In the assignment phase, there are three different constraint rules in the bidding function, which are based on the generalized axiom of revealed preference. The revealed-preference constraint can generally be represented as $\beta(x) \leq \beta(x_t) + p_t(x - x_t)$, where $\beta(x)$ is the bid in the assignment phase for package x, $\beta(x_t)$ is the bid in the clock phase for package x_t, x_t is the package in the clock phase of round t, and p_t is the price in round t. In previous research, three different revealed-preference constraints have been modeled, and the difference between them lies in the clock round used as the baseline [Ausubel and Baranov, 2017][Ausubel and Baranov, 2020].

- The first constraint is the "final cap" rule, which satisfies the revealed preference condition with respect to the final clock round demand. This is denoted as $\beta(x) \leq \beta(\tilde{x}) + \tilde{p}(x - \tilde{x})$.
- The second constraint is the "relative cap" rule, denoted as $\beta(x) \leq \beta(x_t) + p_t(x - x_t)$, where t represents the last round in which the bidder demanded x_t.

- The third constraint, “intermediate cap”, requires bidders’ supplementary bids to be consistent with their behavior during all previous eligibility-reducing rounds. This is denoted as $\beta(x) \leq \beta(x_t) + p_t(x - x_t)$, where $\forall t \in T_{\text{eligibility-reducing}}$

3.2 Utility Functions of Bidders in CCA

After modeling the CCA process, it is important to clarify the properties of multi-unit auctions. Starting with the properties of combinatorial auctions, a key feature distinguishes them from single-unit auctions: in combinatorial auctions, bidders encounter the “complementary goods” issue [Ausubel et al., 2006]. This means that the valuation of a combination of goods, such as (z_1, z_2), is higher than the sum of the valuations of the individual goods, which is known as “synergy effect” [Ausubel et al., 1997]. Therefore, a valuation function $V(x)$ can be constructed, which is strictly increasing and concave in x. It is also assumed that bidder types are randomly drawn from a natural common distribution with the interval $[\underline{\theta}, \bar{\theta}]$. A bidder’s type influences their valuation, with a higher type corresponding to a higher valuation for the same bid bundle. Thus, the function is adjusted to $V(x;\theta)$, where the valuation function is strictly increasing in θ.

Utility is directly derived from valuation, which allows for the proposal of a utility function in the form $U(x;\theta)$. Levin and Skrzypacz [Levin and Skrzypacz, 2016] used a quadratic utility model to represent the utility function over shares during the clock phase. The utility model they used constructed in the form as $U_i(x_i;\theta_i) = V_i(x_i;\theta_i) = \theta_i x_i - \frac{\sigma_i}{2}x_i^2$, where $\theta_i > \sigma_i > 0$. It implies that the marginal valuation is linear, derived as $V_i'(x_i;\theta_i) = \theta_i - \sigma_i x_i$. This will be referred to as the “standard utility function”. Given that demand weakly decreases as price rises and valuation strictly increases with demand, it follows that valuation and utility weakly decreases as price increases.

Revealing prices and packages is one of the key functions of the clock phase. In each round of the clock phase, bidders receive updated information, including their processed demand requests, the new clock price (or whether the clock phase has ended), and the aggregated demand across all bidders. This revealed information enables bidders to estimate their rivals’ types and adjust their bidding strategies accordingly. In an ideal scenario, a completely revealed price during the clock phase satisfies the following objective function:

$$x_i(p) = \arg\max_{x_i}\{U(x_i;\theta_i) - px_i\}$$

When bidder i bids their true valuation, fully disclosing their type, the marginal utility for demand x_i at price p is equal to the current price p, denoted as $U_i'(x_i;\theta_i) = V_i'(x_i;\theta_i) = p$. Additionally, it can be derived that when bidder i has strategic expanding demand behavior, their marginal utility satisfies $U_i'(x_i;\theta_i) = V_i'(x_i;\theta_i) < p$; and when bidder i has strategic reducing demand behavior, their marginal utility satisfies $U_i'(x_i;\theta_i) = V_i'(x_i;\theta_i) > p$.

3.3 Efficiency and Market Equilibrium

For a given type profile, the efficiency of the auction allocation is determined by the optimal allocation outcome $\mathbf{x}^*$, which solves the following maximization problem:

$$\mathbf{x}^* = \arg\max_{x_1,x_2}\{U(x_1;\theta_1) + U(x_2;\theta_2)\}, \text{ s.t. } x_1 + x_2 \leq 1, x_1, x_2 \geq 0$$

Since each bidder’s utility function is weakly increasing with the quantity allocated, and the maximization is a linear combination of their utilities, the optimal allocation will result in a market-clearing condition. In this scenario, the optimal profile satisfies $x_1 + x_2 = 1$. Using the standard utility function, the above objective function can be derived as follows:

$$U(x_1) = (\theta_1 x_1 - \frac{\sigma_1}{2}x_1^2) + (\theta_2(1 - x_1) - \frac{\sigma_2}{2}(1 - x_1)^2)$$

By taking the derivative of the total value, the necessary conditions for efficient allocation (FOCs) as $U_1'(x_1) = U_2'(1-x_1)$. Therefore, the optimal allocation after the clock phase is given by $x_1^* = \frac{\theta_1-\theta_2+\sigma_2}{\sigma_1+\sigma_2}$ and $x_2^* = 1-x_1^* = \frac{\theta_2-\theta_1+\sigma_1}{\sigma_1+\sigma_2}$.

3.4 Strategic Behavior of Bidders

3.4.1 Self-Interested Bidders

Now considering the perspective of the bidder, each bidder faces a tradeoff between the valuation of their won package and the payment for that package. Assuming each bidder bid on their valuation, for bidder i, the final payment is determined by the VCG mechanism, given as $\beta_{-i}(1)-\beta_{-i}(1-x_i)$, where $\beta_{-i}(1) = (\theta_{-i} - \frac{\sigma_{-i}}{2} \times 1^2)$ and $\beta_{-i}(1-x_i) = \theta_{-i}(1-x_i) - \frac{\sigma_{-i}}{2}(1-x_i)^2$. Thus, the payment expression expands to $\theta_{-i}x_i + \frac{\sigma_{-i}}{2}x_i^2 - \sigma_{-i}x_i$. And then the objective function $\arg\max_{x_i}[U_i(x_i;\theta_i) - \text{payoff}_i]$ of the bidder i is rewritten as follows:

$$x_i^n = \arg\max_{x_i}[(\theta_i x_i - \frac{\sigma_i}{2}x_i^2) - (\theta_{-i}x_i + \frac{\sigma_{-i}}{2}x_i^2 - \sigma_{-i}x_i)]$$

To maximize, take the derivative of the objective function with respect to x_i and set it to zero, getting the result as $x_i^n = \frac{\theta_i-\theta_{-i}+\sigma_{-i}}{\sigma_i+\sigma_{-i}}$. This result is consistent with the outcome obtained through overall valuation maximization.

3.4.2 Spiteful Bidders

Janssen M. and Karamychev V. proposed a lexicographics preference to model the spitefulness of bidder on spectrum auction [Janssen and Karamychev, 2016]. When bidder i holds a specific share x_i with a bid $\beta_i(x_i)$, spitefulness may drive them to increase their rival's costs. At this point, bidder i may choose the strategy β_i^* over another strategy β_i based on $\max_y \beta_i^*(y) - \beta_i^*(x_i^*(\theta)) \geq \max_y \beta_{-i}(y) - \beta_{-i}(x_{-i})$.

Although lexicographic preferences provide a direct explanation, this paper adopts a linear utility model for spiteful bidders, inspired by the work of Brandt et al. [Brandt et al., 2007]. The linear utility model offers a significant advantage for analysis, as it allows for numerical methods to be applied effectively instead of relying on qualitative analysis. Moreover, it provides a more effective quantitative tool for empirical identification. The construction of the spiteful bidder i's utility function is as follows:

$$W_i = (1-\alpha_i)[U_i(x_i;\theta_i) - \text{payment}_i] - \alpha_i[U_{-i}(x_{-i};\theta_{-i}) - \text{payment}_{-i}]$$

where $\alpha_i \in [0,1]$ is a parameter called spite coefficient.

Aligned with our assumption that $x_i + x_{-i} = 1$, and based on the analysis from the bidders' perspective, it can be derived that:

$$W_i = (1-\alpha_i)[(\theta_i - \theta_{-i} + \sigma_{-i})x_i - \frac{1}{2}(\sigma_i + \sigma_{-i})x_i^2] - \alpha_i[(\theta_{-i} - \theta_i + \sigma_i)(1-x_i) - \frac{1}{2}(\sigma_i + \sigma_{-i})(1-x_i)^2]$$

For the spiteful bidder, the objective function to solve is as follows:

$$x_i^s = \arg\max_{x_i} W_i$$

Given the FOCs, it can be derived that $x_i^s = \frac{(1-\alpha_i)(\theta_i-\theta_{-i}+\sigma_{-i})+\alpha_i(\theta_{-i}-\theta_i-\sigma_{-i})}{(1-2\alpha_i)(\sigma_i+\sigma_{-i})}$. When a bidder has zero spitefulness, the equilibrium outcome is identical to that of an ideal self-interested bidder. However, in the case of an extreme bidder solely focused on increasing their rivals' costs, the outcome becomes entirely negatively skewed. The existence of equilibrium can now be tested by examining the objective function. Taking the second derivative, the following expression is obtained: $\frac{\partial^2 W}{\partial x_i^2} = (2\alpha_i - 1)(\sigma_i + \sigma_{-i})$. Equilibrium exists only when spitefulness is less than or equal to $\frac{1}{2}$. As spitefulness approaches to $\frac{1}{2}$, the equilibrium becomes unstable.

3.4.3 Strategic Losses

When bidders exhibit spiteful preferences, the equilibrium allocation deviates from the efficiency-maximizing allocation that would arise if all bidders were purely self-interested. Under the baseline model with standard quasi-linear preferences and no spitefulness, the equilibrium allocation x_i^n is constructed to maximize total utility. This ensures that, relative to any other feasible allocation, the chosen x_i^n yields the highest aggregate welfare.

Introducing spitefulness (as captured by the parameters $\alpha_i > 0$) shifts each bidder's equilibrium choice away from this optimal allocation. Because the original allocation x_i^n is a unique and strict maximum of the total utility function, any movement away from it leads to a lower total utility. Thus, the new equilibrium allocation x_i^s, influenced by spiteful motives, necessarily reduces the overall efficiency. In other words, the presence of non-zero spitefulness creates a distortion from the socially optimal solution, and this distortion directly translates into a positive efficiency loss.

3.5 Strategy Spaces in CCA

3.5.1 Feasibility of the Clock Phase

Assuming the bidder follows a truth-telling strategy, each round will reflect their valuation at the given price. As the price rises, each bidder's demand will weakly decrease from full demand (1) to the final demand $\tilde{x}$. In each round, bidder i demands x_i units at a price where $V_i'(x_i) = p$. The total cost of the package can then be represented as $x_i V_i'(x_i)$. Assume that on the final clock, bidder i submits a demand $\tilde{x}_i$ at price $\tilde{p}$. The share of the rival is then given by $\tilde{x}_{-i} = 1 - \tilde{x}_i$.

Although the assumption of truth-telling is ideal, bidders often adopt strategic bidding behavior during the clock phase. As shown earlier, the auctioneer aims for overall maximization, where $U_1'(x_1;\theta_1) = U_2'(x_2;\theta_2)$. Assume a two-bidder auction scenario involving one spiteful bidder and one self-interested bidder. In the early phase of the clock stage, a spiteful bidder may expand their demand such that $U_1'(x_1;\theta_1) < p$. This means a bidder bids on a higher marginal valuation than their true marginal valuation. From another perspective, this behavior allows the bidder to push the observed clock price above their real marginal utility. In this case, market clearing occurs when $\hat{U}_1'(x_1;\theta_1) = U_2'(x_2;\theta_2)$, where $\hat{U}_1'(x_1;\theta_1) > U_1'(x_1;\theta_1)$. Given $U_i' = \theta_i - \sigma_i x_i$, it follows that $\hat{\theta}_i = \hat{U}_i' + \hat{\sigma}_i x_i = p + \hat{\sigma}_i x_i$, which explains how a bidder can estimate the type of their rivals. Therefore, when a spiteful bidder expands their demand during the clock phase, their rivals are likely to overestimate their type.

The feasibility of spiteful bidding can be illustrated through a scenario involving two asymmetric bidders. Suppose bidder 1 has a higher type, such that $\theta_1 > \theta_2$. In an observed (but artificial) dynamic market, the price is given by $p_t = \theta_i - \sigma_i x_i$. Holding σ_i and x_i fixed, an increase in θ_i leads to a higher price. The intuition behind this is that demand expansion creates an illusion of a more competitive market environment. Although the clock phase still concludes with a price close to the second-highest valuation, this second-highest valuation is distorted and artificially inflated. However, spiteful bidding is not without risk: misestimating rivals may lead to a dead end, and the spiteful bidder may secure the license they do not actually desire.

If bidder i has an interval for their expected package amounts, they can accept the clock phase ending with market clearing for any possible price $p \in [U_i'(\bar{x}_i), U_i'(\underline{x}_i)]$. As the final clock price falls within this interval, in the assignment phase they will focus on raising the price that their competitor has to pay. This explains why weaker types have an incentive to deviate and expand their demand, at least until the clock price reaches the interior of this interval. At that point, they drop their demand discontinuously and bid truthfully thereafter.

As of now, since bidders reveal their types during the clock phase, it follows that under a weak preference for raising rivals' costs, the CCA cannot deliver an efficient outcome. Specifically, a stronger bidder tends to hide their true valuation during the clock phase, aiming to secure a lower cost in the assignment phase. Meanwhile, a weaker bidder may observe that if the clock phase has not ended, it is likely that their rivals have a higher type. This observation incentivizes the weaker bidder to place more aggressive bids, thereby driving up their rivals' costs.

3.5.2 Feasibility of the Assignment Phase

After the clock phase concludes, each bidder i has revealed a final demand $\tilde{x}_i$ and faces a final clock price $\tilde{p}$. The feasible bidding space in the assignment phase is now defined. For simplicity, consider that the bidder is effectively placing a single supplementary bid for the entire package corresponding to the final demand identified in the clock phase. This supplementary bid, denoted by $\beta_i(x_i)$ for any chosen quantity x_i, will be subject to both a lower and an upper bound.

The minimum bid rule specifies that a bidder's supplementary bid must not fall below the product of the final clock price and the desired quantity. Formally, for any package size x_i under consideration, it holds that $\beta_i(x_i) \geq \tilde{p}x_i$. Intuitively, this ensures that the bidder does not drastically understate their valuation relative to the signals revealed during the clock phase.

Determining the upper bound on supplementary bids is more nuanced. The analysis is based on the generalized axiom of revealed preference (GARP), which guides three specific types of constraints that were introduced earlier:

- When the CCA adopts the "final cap" rule, it uses the final clock round $(\tilde{x}_i, \tilde{p})$ as a reference point. This rule effectively enforces a linear upper bound based on the final clock price and allocation.
- When the CCA adopts the "relative cap" rule, it uses the last clock round in which the bidder demanded a particular quantity x_t. This rule provides an upper bound based on a previously revealed point (x_t, p_t). Unlike the final cap, which relies solely on the last clock round, the relative cap can draw on different points during the clock phase, potentially resulting in tighter constraints.
- Finally, when the CCA adopts the "intermediate cap" rule, this rule aggregates constraints from all eligibility-reducing rounds. By considering multiple baseline points, the intermediate cap provides an even stricter upper bound. The effective upper limit is then determined by the lowest of these linear constraints across all eligibility-reducing rounds.

Regardless of which cap rule is chosen, the activity rules provide a strategic space for spiteful bidders. A spiteful bidder can evaluate the possible bidding interval of their rival based on these limits. This allows the spiteful bidder to place a higher, but not excessively high, bid to raise their rival's cost while ensuring that items they do not desire are not won. To illustrate this intuition, an example is presented below.

When considering a "naive" spiteful bidder—one who assumes that their rival is bidding truthfully in accordance with a VCG-like strategy—the key challenge lies in inferring the rival's valuation parameters from observed outcomes in the clock phase, and then using these parameters to estimate the rival's overall valuation function.

First, note that if the naive spiteful bidder believes the rival's bidding behavior in the clock phase truthfully reflects their underlying valuation, the final clock price and demand provide a direct signal of the rival's marginal valuation at that specific point. For example, if the clock phase concludes with the rival holding a share $\tilde{x}_{-i}$ at final price $\tilde{p}$, under truthful bidding this final price would satisfy $\tilde{p} = V'_{-i}(\tilde{x}_{-i}) = \theta_{-i} - \sigma_{-i}\tilde{x}_{-i}$. Hence, given a known or estimated σ_{-i}, the naive spiteful bidder can infer the rival's type parameter as $\hat{\theta}_{-i} = \tilde{p} + \sigma_{-i}\tilde{x}_{-i}$.

However, knowing the rival's type $\hat{\theta}_{-i}$ alone does not immediately translate into a complete picture of their valuations. The type parameter is not itself the valuation, but rather a key input into the known functional form of the rival's valuation function. Since valuations are modeled using a parametric function such as $V_{-i}(x;\theta_{-i}) = \theta_{-i}x - \frac{\sigma_{-i}}{2}x^2$, substituting the inferred $\hat{\theta}_{-i}$ back into this functional form reconstructs the entire valuation curve for any quantity x. Thus, once the naive spiteful bidder obtains an estimate of $\hat{\theta}_{-i}$ from the final clock data, they can derive the rival's valuation for any bundle size, facilitating more precise strategic decision-making in the assignment phase.

This estimation process enables the spiteful bidder to better understand the rival's willingness to pay and to carefully position their own bid within the allowable bounds—determined by the activity rules and GARP constraints—thereby raising the rival's costs without inadvertently securing unwanted goods.

4 Empirical Identification

When addressing the issue of identification, it is essential to establish a clear link between the observations and the theory. The CCA environment provides several sources of data, and the goal is to use this data to identify whether bidders behave strategically (spiteful bidding) and to estimate their underlying demand curve parameters (θ_i, σ_i) and spitefulness parameter α_i.

4.1 Data Collection

First, for the clock phase of the CCA, the bidding demands for each item from each bidder can be obtained. This can be denoted as $\mathbf{x}_{it} = \{x_{1_{it}}, x_{2_{it}}, \dots, x_{n_{it}}\}$, where $\mathbf{x}_{it}$ represents the demand vector for bidder bidder i in round t. Specifically, for bidder i in round t, $x_{1_{it}}$ represents their bid on item 1, $x_{2_{it}}$ represents their bid on item 2, and so forth. Similarly, the associated price vector for bidder i in round t, denoted as $\mathbf{p}_{it}$, can also be obtained. Since the prices are unique within round t, the price vector can be simplified as $\mathbf{p}_t$. the final status of the clock phase can be determined, including the final price vector $\tilde{\mathbf{p}}$ and the final demand vector $\tilde{\mathbf{x}}_i$ for each bidder.

Second, for the assignment phase of the CCA, the bidding package of each bidder can be obtained, denoted again as $\mathbf{x}_i$. The only difference is that the assignment phase consists of a single round, so the time index t is no longer needed. In this phase, each bidder submits a total valuation for the package, denoted as $\beta_i(\mathbf{x}_i)$.

4.2 Identification Strategy

First, bidder behavior in the clock phase is examined. Demand adjustments during the clock phase directly reflect how bidders respond to rising prices. To understand this relationship, a non-parametric approach could initially be considered to estimate aggregate demand as a function of price:

$$\hat{Q}(p) = \frac{\sum_{i=1}^{n} K\left(\frac{p-P_i}{h}\right) Q_i}{\sum_{i=1}^{n} K\left(\frac{p-P_i}{h}\right)}$$

where $K(\cdot)$ is a kernel function and h is a bandwidth parameter. This non-parametric estimation allows for the observation of general patterns of demand changes without imposing a specific functional form.

However, to rigorously identify bidder-specific parameters (θ_i, σ_i), the linear structure implied by the theoretical model is leveraged. Under truthful (or at least consistent) bidding behavior, bidder i's equilibrium condition in each round t is given by:

$$x_i = \frac{\theta_i - p_t}{\sigma_i}$$

Given multiple rounds of observed (p_t, x_i), this system provides sufficient variation to uniquely identify θ_i and σ_i using linear regression. Specifically, by rewriting the demand equation as:

$$p_t = \theta_i - \sigma_i x_i$$

θ_i is estimated as the intercept and σ_i as the slope of the linear regression. This provides a direct parametric identification of each bidder's underlying demand curve parameters.

4.3 Addressing Strategic Behaviors

However, strategic bidding behaviors such as demand expansion (overstating x_i) or demand reduction (understating x_i) during the clock phase could distort the observed data. To address this issue, revealed preference constraints are imposed to ensure consistency between observed behaviors and true preferences [Kroemer et al., 2017]. Only those bidding patterns that remain consistent with rational, preference-revealing behavior are retained for parameter estimation. This ensures that demand data reflecting true underlying preferences, rather than strategic distortions, is utilized.

The core idea of classical GARP is revealed preference and transitivity. By comparing the choices of bidders under different budget conditions, it is possible to determine whether their behavior is consistent with utility maximization. For the CCA, bid information during the clock phase can be obtained as $(\mathbf{x}_{it}, \mathbf{p}_{it})$, where $\mathbf{x}_{it}$ represents the demand vector of bidder i in round t, and $\mathbf{p}_{it}$ represents the price vector of bidder i in round t. Therefore, the expenditure can be compared by calculating the dot product between the demand vector and the price vector. Specifically, $\mathbf{x}_{it_1} \cdot \mathbf{p}_{it_1}$, $\mathbf{x}_{it_2} \cdot \mathbf{p}_{it_2}$, $\mathbf{x}_{it_1} \cdot \mathbf{p}_{it_2}$ and $\mathbf{x}_{it_2} \cdot \mathbf{p}_{it_1}$ are constructed, where t_1 and t_2 denote two distinct rounds. If $\mathbf{x}_{it_1} \cdot \mathbf{p}_{it_2} < \mathbf{x}_{it_1} \cdot \mathbf{p}_{it_1}$ and $\mathbf{x}_{it_2} \cdot \mathbf{p}_{it_1} < \mathbf{x}_{it_2} \cdot \mathbf{p}_{it_2}$, then there is a strictly direct revealed preference violation. Otherwise, the direct revealed preference condition holds for these two observations. GARP extends beyond pairwise comparisons to ensure no cycles arise across multiple observations. Consider a set of price-demand pairs $\{(\mathbf{p}_{it_1}, \mathbf{x}_{it_1}), (\mathbf{p}_{it_2}, \mathbf{x}_{it_2}), \dots, (\mathbf{p}_{it_n}, \mathbf{x}_{it_n})\}$, if there exists a sequence of bundles such that:

$$\mathbf{p}_{it_1} \cdot \mathbf{x}_{it_1} \leq \mathbf{p}_{it_1} \cdot \mathbf{x}_{it_2}, \quad \mathbf{p}_{it_2} \cdot \mathbf{x}_{it_2} \leq \mathbf{p}_{it_2} \cdot \mathbf{x}_{it_3}, \dots, \quad \mathbf{p}_{it_{n-1}} \cdot \mathbf{x}_{it_{n-1}} \leq \mathbf{p}_{it_{n-1}} \cdot \mathbf{x}_{it_n}$$

then GARP requires that:

$$\mathbf{p}_{it_1} \cdot \mathbf{x}_{it_1} \leq \mathbf{p}_{it_1} \cdot \mathbf{x}_{it_n}$$

By filtering out any data that do not satisfy these revealed preference conditions, the analysis is narrowed to a set of observations that likely reflect true underlying preferences. This filtered dataset can then be used to run the linear regression for (θ_i, σ_i), yielding more reliable parameter estimates.

4.4 Incorporating Supplementary Bids

Finally, the sealed-bid data from the assignment phase is incorporated. These bids are presumed closer to true valuations since the assignment phase is designed to incentivize truthful bidding. By comparing the derived parameters (θ_i, σ_i) and checking whether $U_i'(x_i) = p_t$ holds under these less-distorted conditions, the identification strategy is further validated. If the parameters estimated from filtered clock-phase data align with valuations implied by supplementary bids, it strengthens the credibility of our estimated parameters and the overall identification approach.

4.5 Measuring Spiteful Bidding

If the objective is to identify the spitefulness parameter α_i, the key lies in assessing whether allowing for spiteful preferences improves the rationalization of observed bidding behavior. Initially, the parameters (θ_i, σ_i)

and $(\theta_{-i}, \sigma_{-i})$ are estimated under the assumption of purely self-interested behavior, employing GARP-based screening to ensure that only preference-consistent observations enter the estimation. Supplementary sealed bids are then used to further validate these baseline parameters.

Once estimates of both the bidder's and the rival's valuation parameters are obtained, the rival's net utility function can be reconstructed, allowing the derivation of "shadow prices" to reflect how changes in the bidder's own allocation affect the rival's payoff. Incorporating these shadow prices leads to the construction of an adjusted price vector:

$$\tilde{\mathbf{p}}_{it} = (1 - \alpha_i)\mathbf{p}_{it} - \alpha_i \mathbf{r}_{it},$$

where $\mathbf{r}_{it}$ represents the marginal impact on the rival's utility. $\mathbf{r}_{it}$ can be estimated using the following equation:

$$\mathbf{r}_{it} = \hat{\theta}_{-i} - \hat{\sigma}_{-i}\mathbf{x}_{it}$$

If the data cannot be rationalized under the assumption $\alpha_i = 0$, it is possible to incrementally increase α_i and reapply the GARP test using the adjusted price vectors. Identifying a positive $\hat{\alpha}_i$ that eliminates previously observed preference inconsistencies implies that introducing spitefulness enhances the explanatory power of the model. In this manner, $\hat{\alpha}_i$ can be inferred as the value of the spitefulness parameter that best rationalizes the observed bidding patterns within a spiteful preference framework.

5 Field Evidence

5.1 Overview of the Auction 107

For the theory discussed above, it is essential to confirm whether there is field evidence in the real world. The FCC has held a series of spectrum auctions using the CCA format, and they publish the auction rules and data for each auction on their official website. Although there is heterogeneity among these auctions and bidders' strategies vary, specific auctions provide opportunities to analyze the existence of strategic behaviors and the motivations behind spiteful bidding. Auction 107, in particular, has been suspected of involving spiteful bidding behaviors. This auction will be analyzed in detail in the following sections.

The analysis of Auction 107 begins with a summary of the auction data. In Auction 107, 57 companies were qualified to participate, and 5,684 licenses were available for sale. The auction consisted of a total of 97 rounds during the clock phase. At the end of the auction, 21 bidders won all 5,684 licenses, achieving market clearing. The auction generated a total gross income of $81,168,677,645, which was significantly higher than previous estimates by auction analysts.

Table 1: Bidding Statistics for Different Rounds in the Auction

	Round	Number of Bidders	Aggregated Quantity	Average Quantity
First Round	1	7808	44	177.4545
Highest Aggregated Quantity	23	11094	41	270.5854
Highest Average Quantity	38	9402	32	293.8125
Last Round	97	5458	21	259.9048

Summarizing specific rounds can provide a quick overview of the auction. Table 1 presents the bidder count, aggregated demand, and average demand per bidder for four specific rounds: the initial round, the final round, the round with the highest aggregated demand, and the round with the highest average demand. Based on the data in Table 1, evidence of demand expansion can be observed in Auction 107. The highest aggregated demand occurred during the early stages of the clock phase, approximately 1.4 times that of the initial round. Similarly, the highest average demand also occurred in the early stages, exceeding 1.6 times that of the initial round.

5.2 Insights from the Auction 107

After using non-parametric estimation to analyze the demand changes of bidders in response to price increases, some interesting evidence has emerged. Different patterns of demand changes were observed, suggesting that bidders employed varying bidding strategies.

The first type of bidders can be referred to as “truth-tellers”, as their demand consistently decreased as prices rose. This behavior aligns with their true marginal valuations, indicating that they bid truthfully during the clock phase.

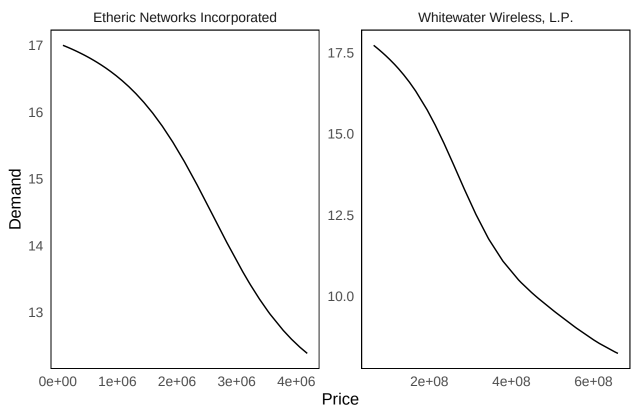

Figure 1. The plot illustrates the demand curves of two truth−tellers.

The second type of bidders can be referred to as “strategic players”. Unlike truth-tellers, their demand curves do not follow the expected pattern. Instead, they exhibit unusual bidding behaviors, such as sudden decreases in demand or even increases in demand as prices rise. These records suggest that their bidding may be driven by strategic motivations, including spitefulness.

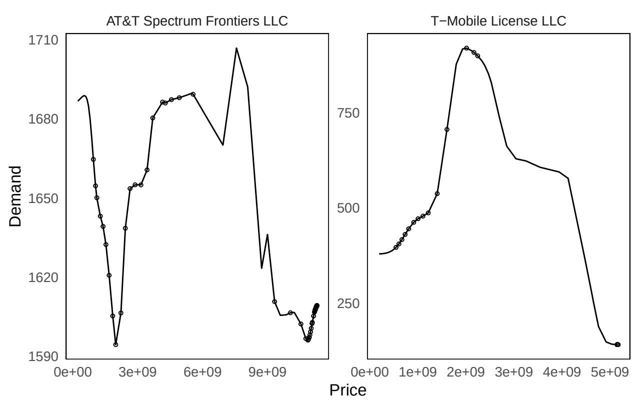

Figure 2. The plot illustrates the demand curves of two strategic players. The cycles shown on the plot indicate the rounds in which the bidder´s bids violated GARP.

After the analysis, most bidders exhibited bidding behaviors that violated GARP, with 23 bidders violating WARP and 43 violating GARP. Moreover, all of the winners displayed bidding behaviors that violated the GARP. This implies that spiteful bidding behaviors were prevalent in the auction, replacing truth-telling as the dominant strategy.

Now, let T-Mobile License LLC, the spiteful player, serve as an example. Using the identification method provided in the previous section, each round of the company in the clock phase is verified to determine

Table 2: Estimated Parameters of T-Mobile License LLC and Its Rivals in the Clock Phase

	T-Mobile License LLC			Rivals		
Statistic	Estimate	Std. Error	p-value	Estimate	Std. Error	p-value
Intercept (θ_i)	25.5652	0.7517	$< 2 \times 10^{-16}$	24.3068	0.7668	$< 2 \times 10^{-16}$
Slope (σ_i)	0.6849	0.1358	3.16×10^{-6}	0.3648	0.1066	0.000922
Observations	75			97		
R-squared	0.256			0.110		
Adjusted R-squared	0.246			0.100		
Residual Std. Error	0.853 (df = 74)			0.920 (df = 95)		
F-statistic	25.43 ($p = 3.16 \times 10^{-6}$)			11.7 ($p = 0.000922$)		

whether it violated the GARP. After this verification, the parameters of its standard utility function are evaluated, as shown in Table 2. For the rivals of T-Mobile License LLC, using the aggregated quantity for calculation is reasonable. During the auction, the auctioneer only publicly reveals the aggregated quantity of each round while concealing individual rivals' information until the end of the auction. This aims to eliminate information that could be exploited for strategic behavior. Therefore, in the estimation process, the aggregated quantity is used to construct an overall model for the rivals. The estimated parameters of the rivals are also presented in Table 2. Both estimations use a log-to-log linear regression model to account for scale size differences in the estimation process.

By plugging the estimated parameters into the iterative algorithm, the level of spitefulness is estimated to be approximately 0.35. This means that, assuming T-Mobile License LLC has a spitefulness level of 0.35, the company's behavior would no longer violate GARP under the spiteful model outlined in the previous section.

It appears that T-Mobile License LLC exhibited significant spitefulness. In fact, this aligns with perspectives shared by some auction analysts and news reporters. Furthermore, it has been confirmed that T-Mobile License LLC was estimated to have substantially increased the costs for its rivals. As discussed in the first section, spiteful bidding behavior can heavily strain the financial budgets of rivals. As predicted, in the subsequent auction (Auction 108), the rivals of T-Mobile License LLC remained relatively quiet and lost most of the licenses.

6 Conclusion

This paper offers an explanation for demand expansion in the combinatorial clock auction (CCA), arguing that bidders' motivations extend beyond the valuation of their chosen packages to include potential incentives to increase rivals' costs. It further examines how the CCA provides strategic opportunities for spiteful bidders. Specifically, the clock phase reveals rival types through the price revelation process, while the simultaneous achievement of package revelation and effective equilibrium remains unattainable.

Additionally, this study contributes to the literature by proposing a novel framework for identifying spiteful bidding behavior in practice. The framework leverages GARP to detect deviations from optimal bidding behavior and employs available data to estimate bidders' utility function parameters. An extended GARP framework is also introduced to specifically identify spiteful motivations. Finally, empirical evidence from FCC Auction 107 substantiates the theoretical claims, completing the empirical verification of the proposed theories.

Nonetheless, it is important to acknowledge that the utility function adopted in this study may not fully capture bidders' real preferences. Factors such as budget constraints [Janssen et al., 2017] and evaluation biases could further influence bidding strategies. Moreover, the heterogeneity of auctions may constrain the explanatory power of the field evidence presented. Addressing these limitations should be a priority for future research.

7 Appendix

7.1 Equilibrium Analysis

1. Problem Setup Review:
The standard utility functions for two bidders are given as:

$$U_i(x_i; \theta_i) = \theta_i x_i - \frac{\sigma_i}{2} x_i^2.$$

Given the type configuration (θ_1, θ_2), the goal is to solve the optimal allocation problem that maximizes total utility while satisfying the market-clearing condition:

$$\max_{x_1, x_2} U(x_1; \theta_1) + U(x_2; \theta_2) \quad \text{s.t. } x_1 + x_2 \leq 1,\ x_1, x_2 \geq 0.$$

Since each bidder's utility function is increasing (when the marginal utility is positive) and concave in their own allocation, the optimal solution will typically "fill" the total available resource, implying $x_1 + x_2 = 1$. Thus, x_2 can be eliminated using the constraint $x_2 = 1 - x_1$.

2. Total Objective Function:
Replacing x_2 with $1 - x_1$, the total utility becomes:

$$U(x_1; \theta_1) + U(1 - x_1; \theta_2) = (\theta_1 x_1 - \tfrac{\sigma_1}{2} x_1^2) + \left[\theta_2(1 - x_1) - \tfrac{\sigma_2}{2}(1 - x_1)^2\right].$$

Expanding the second term:

$$\begin{aligned} U(x_2; \theta_2) &= \theta_2(1 - x_1) - \tfrac{\sigma_2}{2}(1 - 2x_1 + x_1^2) \\ &= \theta_2 - \theta_2 x_1 - \tfrac{\sigma_2}{2}(1 - 2x_1 + x_1^2) \\ &= \theta_2 - \theta_2 x_1 - \tfrac{\sigma_2}{2} + \sigma_2 x_1 - \tfrac{\sigma_2}{2} x_1^2. \end{aligned}$$

Combining terms:

- Constant term: $\theta_2 - \frac{\sigma_2}{2}$
- Linear term in x_1: $(-\theta_2 x_1 + \sigma_2 x_1)$ combines to $(\sigma_2 - \theta_2)x_1$
- Quadratic term in x_1: $-\frac{\sigma_2}{2} x_1^2$

Thus:

$$U(1 - x_1; \theta_2) = \theta_2 - \frac{\sigma_2}{2} + (\sigma_2 - \theta_2)x_1 - \frac{\sigma_2}{2} x_1^2.$$

Now adding $U(x_1; \theta_1)$:

$$U(x_1; \theta_1) = \theta_1 x_1 - \frac{\sigma_1}{2} x_1^2.$$

Summing both terms:

$$U_{\text{total}}(x_1) = \left[\theta_1 x_1 - \frac{\sigma_1}{2} x_1^2\right] + \left[\theta_2 - \frac{\sigma_2}{2} + (\sigma_2 - \theta_2)x_1 - \frac{\sigma_2}{2} x_1^2\right].$$

Combining like terms:

- Constant term: $\theta_2 - \frac{\sigma_2}{2}$
- Linear term in x_1: $\theta_1 x_1 + (\sigma_2 - \theta_2)x_1 = (\theta_1 - \theta_2 + \sigma_2)x_1$
- Quadratic term in x_1: $-\frac{\sigma_1}{2}x_1^2 - \frac{\sigma_2}{2}x_1^2 = -\frac{\sigma_1+\sigma_2}{2}x_1^2$

Final expression:

$$U_{\text{total}}(x_1) = \theta_2 - \frac{\sigma_2}{2} + (\theta_1 - \theta_2 + \sigma_2)x_1 - \frac{\sigma_1 + \sigma_2}{2}x_1^2.$$

3. First-Order Condition (FOC):
Taking the derivative with respect to x_1 and setting it to zero to find the maximizer:

$$\frac{dU_{\text{total}}}{dx_1} = (\theta_1 - \theta_2 + \sigma_2) - (\sigma_1 + \sigma_2)x_1 = 0.$$

Solving for x_1^*:

$$(\sigma_1 + \sigma_2)x_1 = \theta_1 - \theta_2 + \sigma_2 \implies x_1^* = \frac{\theta_1 - \theta_2 + \sigma_2}{\sigma_1 + \sigma_2}.$$

Thus, x_2 can be eliminated using the constraint $x_2 = 1 - x_1$:

$$x_2^* = 1 - x_1^* = 1 - \frac{\theta_1 - \theta_2 + \sigma_2}{\sigma_1 + \sigma_2}.$$

For symmetry, write:

$$x_2^* = \frac{\theta_2 - \theta_1 + \sigma_1}{\sigma_1 + \sigma_2}.$$

7.2 Optimization Strategies for Self-Interested Bidders

1. Problem Description Review:
From the perspective of bidder i, their goal is to maximize their net utility, i.e., utility minus payment. Under the VCG mechanism, the final payment for bidder i is the difference between the optimal revenue of the opponents without i and the remaining revenue of the opponents with i.

The payment formula is:

$$\text{Payment}_i = \beta_{-i}(1) - \beta_{-i}(1 - x_i).$$

Where:

$$\beta_{-i}(1) = \theta_{-i} - \frac{\sigma_{-i}}{2} \cdot 1^2 = \theta_{-i} - \frac{\sigma_{-i}}{2}.$$

$$\beta_{-i}(1 - x_i) = \theta_{-i}(1 - x_i) - \frac{\sigma_{-i}}{2}(1 - x_i)^2.$$

2. Computing the Payment Term:
Calculate Payment_i:

$$\text{Payment}_i = [\theta_{-i} - \tfrac{\sigma_{-i}}{2}] - [\theta_{-i}(1 - x_i) - \tfrac{\sigma_{-i}}{2}(1 - 2x_i + x_i^2)].$$

Expand the second bracket:

$$\beta_{-i}(1-x_i) = \theta_{-i}(1-x_i) - \frac{\sigma_{-i}}{2}(1-2x_i+x_i^2).$$

Further expanding:

$$\beta_{-i}(1-x_i) = \theta_{-i} - \theta_{-i}x_i - \frac{\sigma_{-i}}{2} + \sigma_{-i}x_i - \frac{\sigma_{-i}}{2}x_i^2.$$

Combining terms:

$$\beta_{-i}(1-x_i) = \theta_{-i} - \frac{\sigma_{-i}}{2} + (\sigma_{-i}-\theta_{-i})x_i - \frac{\sigma_{-i}}{2}x_i^2.$$

Thus, the payment becomes:

$$\text{Payment}_i = (\theta_{-i} - \tfrac{\sigma_{-i}}{2}) - [\theta_{-i} - \tfrac{\sigma_{-i}}{2} + (\sigma_{-i}-\theta_{-i})x_i - \tfrac{\sigma_{-i}}{2}x_i^2].$$

Simplify the bracket:

$$\text{Payment}_i = \theta_{-i} - \frac{\sigma_{-i}}{2} - \theta_{-i} + \frac{\sigma_{-i}}{2} - (\sigma_{-i}-\theta_{-i})x_i + \frac{\sigma_{-i}}{2}x_i^2.$$

The constant terms cancel out:

$$\text{Payment}_i = -(\sigma_{-i}-\theta_{-i})x_i + \frac{\sigma_{-i}}{2}x_i^2.$$

Rewriting:

$$\text{Payment}_i = (\theta_{-i}-\sigma_{-i})x_i + \frac{\sigma_{-i}}{2}x_i^2.$$

This matches our simplified result, $(\theta_{-i}x_i + \frac{\sigma_{-i}}{2}x_i^2 - \sigma_{-i}x_i)$, as $\theta_{-i}x_i - \sigma_{-i}x_i = (\theta_{-i}-\sigma_{-i})x_i$.

3. Bidder i's Objective Function:
The net utility for bidder i is:

$$\begin{aligned} U_i(x_i;\theta_i) - \text{Payment}_i &= (\theta_i x_i - \frac{\sigma_i}{2}x_i^2) - [(\theta_{-i}-\sigma_{-i})x_i + \frac{\sigma_{-i}}{2}x_i^2] \\ &= (\theta_i - \theta_{-i} + \sigma_{-i})x_i - \frac{\sigma_i}{2}x_i^2 - \frac{\sigma_{-i}}{2}x_i^2 \\ &= (\theta_i - \theta_{-i} + \sigma_{-i})x_i - \frac{\sigma_i + \sigma_{-i}}{2}x_i^2. \end{aligned}$$

4. First-Order Condition (FOC):
Take the derivative with respect to x_i:

$$\frac{d}{dx_i}[(\theta_i - \theta_{-i} + \sigma_{-i})x_i - \frac{\sigma_i + \sigma_{-i}}{2}x_i^2] = (\theta_i - \theta_{-i} + \sigma_{-i}) - (\sigma_i + \sigma_{-i})x_i.$$

Set this to zero to find the maximizer:

$$(\theta_i - \theta_{-i} + \sigma_{-i}) - (\sigma_i + \sigma_{-i})x_i = 0.$$

$$x_i = \frac{\theta_i - \theta_{-i} + \sigma_{-i}}{\sigma_i + \sigma_{-i}}.$$

7.3 Optimization Strategies for Spiteful Bidders

1. Problem Setup Review:
The utility function for a spiteful bidder is a linear combination of their own net utility (valuation minus payment) and their opponent's net utility (opponent's valuation minus payment). The degree of spitefulness is reflected by the parameter α_i, the spite coefficient:

$$W_i = (1 - \alpha_i)[U_i(x_i; \theta_i) - \text{payment}_i] - \alpha_i[U_{-i}(x_{-i}; \theta_{-i}) - \text{payment}_{-i}],$$

where $\alpha_i \in [0, 1]$.

- When $\alpha_i = 0$, the bidder cares only about their own net utility, reducing the problem to the previous individual optimization case.
- When $\alpha_i = 1$, the bidder only cares about reducing the opponent's net utility, representing extreme spitefulness.

According to the earlier setup, $x_i + x_{-i} = 1$, and the standard utility function is $U_i(x_i; \theta_i) = \theta_i x_i - \frac{\sigma_i}{2} x_i^2$. The payment structure follows the VCG mechanism:

$$\text{payment}_i = \beta_{-i}(1) - \beta_{-i}(1 - x_i),$$

which, in the standard model, simplifies to:

$$\text{payment}_i = (\theta_{-i} - \sigma_{-i})x_i + \frac{\sigma_{-i}}{2} x_i^2.$$

The opponent's net utility is similarly defined, and $x_{-i} = 1 - x_i$.

2. Expansion of the Spiteful Utility Function:
Substitute $U_i(x_i; \theta_i) - \text{payment}_i$ and $U_{-i}(x_{-i}; \theta_{-i}) - \text{payment}_{-i}$ into W_i.

Based on our previous analysis, the bidder's own net utility is:

$$U_i(x_i; \theta_i) - \text{payment}_i = (\theta_i - \theta_{-i} + \sigma_{-i})x_i - \frac{\sigma_i + \sigma_{-i}}{2} x_i^2.$$

The opponent's net utility (using $x_{-i} = 1 - x_i$) is:

$$U_{-i}(x_{-i}; \theta_{-i}) - \text{payment}_{-i} = (\theta_{-i} - \theta_i + \sigma_i)(1 - x_i) - \frac{\sigma_i + \sigma_{-i}}{2}(1 - x_i)^2.$$

Substituting these into W_i:

$$W_i = (1 - \alpha_i)[(\theta_i - \theta_{-i} + \sigma_{-i})x_i - \frac{\sigma_i + \sigma_{-i}}{2} x_i^2] - \alpha_i[(\theta_{-i} - \theta_i + \sigma_i)(1 - x_i) - \frac{\sigma_i + \sigma_{-i}}{2}(1 - x_i)^2].$$

The first term can be expanded as follows:

$$(1 - \alpha_i)(\theta_i - \theta_{-i} + \sigma_{-i})x_i - (1 - \alpha_i)\frac{\sigma_i + \sigma_{-i}}{2} x_i^2$$

The second term can be expanded as follows:

$$-\alpha_i(\theta_{-i} - \theta_i + \sigma_i) + \alpha_i(\theta_{-i} - \theta_i + \sigma_i)x_i + \alpha_i\frac{\sigma_i + \sigma_{-i}}{2} + \alpha_i\frac{\sigma_i + \sigma_{-i}}{2} x_i^2 - \alpha_i\frac{\sigma_i + \sigma_{-i}}{2} 2x_i$$

Substituting these into W_i, the x_i term becomes:

$$[(1-\alpha_i)(\theta_i - \theta_{-i} + \sigma_{-i}) + \alpha_i(\theta_{-i} - \theta_i + \sigma_i) - \alpha_i(\sigma_i + \sigma_{-i})]x_i$$

the x_i^2 term becomes:

$$-[(1-\alpha_i)\frac{\sigma_i + \sigma_{-i}}{2} - \alpha_i \frac{\sigma_i + \sigma_{-i}}{2}]x_i^2$$

the constant term is expressed as:

$$-\alpha_i(\theta_{-i} - \theta_i + \sigma_i) + \alpha_i \frac{\sigma_i + \sigma_{-i}}{2}$$

3. First-Order Condition (FOC):
Take the derivative of W_i with respect to x_i and set it to zero to find the optimal decision x_i^s.

The result is:

$$x_i^s = \frac{(1-\alpha_i)(\theta_i - \theta_{-i} + \sigma_{-i}) + \alpha_i(\theta_{-i} - \theta_i - \sigma_{-i})}{(1-2\alpha_i)(\sigma_i + \sigma_{-i})}.$$

Verify extreme cases:

- If $\alpha_i = 0$ (no spite):

  $$x_i^s = \frac{\theta_i - \theta_{-i} + \sigma_{-i}}{\sigma_i + \sigma_{-i}} = x_i^n,$$

  which matches the neutral (self-interested) equilibrium result.

4. Second-Order Condition and Equilibrium Existence:
The second derivative is:

$$\frac{\partial^2 W_i}{\partial x_i^2} = (2\alpha_i - 1)(\sigma_i + \sigma_{-i}).$$

- If $\alpha_i < \frac{1}{2}$, $(2\alpha_i - 1) < 0$, and since $(\sigma_i + \sigma_{-i}) > 0$, the second derivative is negative, making the problem strictly concave, ensuring a unique optimal solution.

- When $\alpha_i = \frac{1}{2}$, the second derivative is zero, resulting in a boundary case with unstable equilibrium.

- If $\alpha_i > \frac{1}{2}$, the second derivative is positive, making the utility function convex in x_i, with no stable interior maximum (potentially leading to boundary solutions). The equilibrium is unstable or nonexistent.

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